Bézier Circular Arc Parameter

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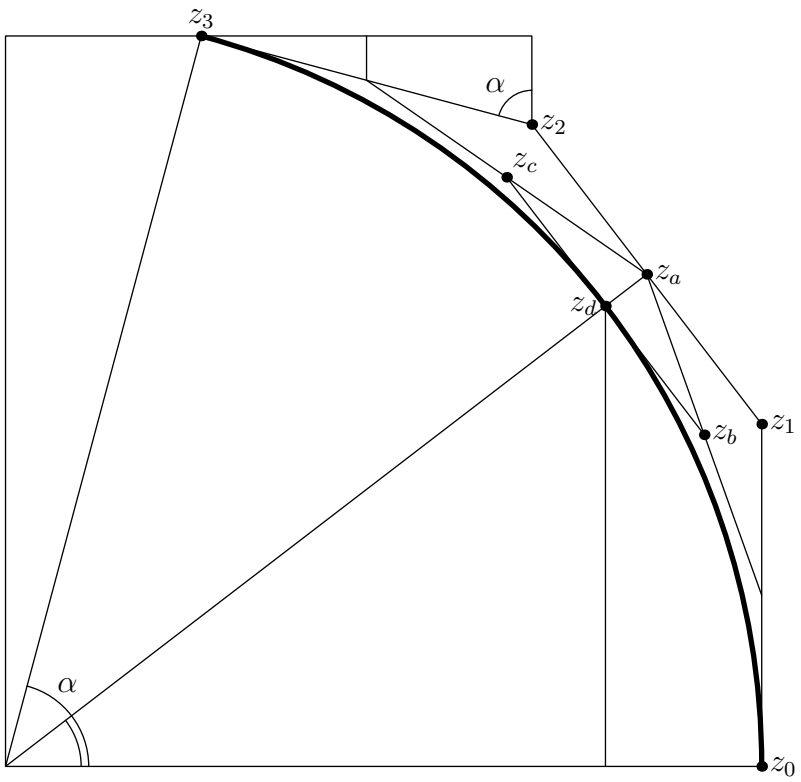
1 Derivation

Bézier curve spanned from z_0 to z_3, control points z_1 and z_2. See `curveto` operator, page 393 in [1].

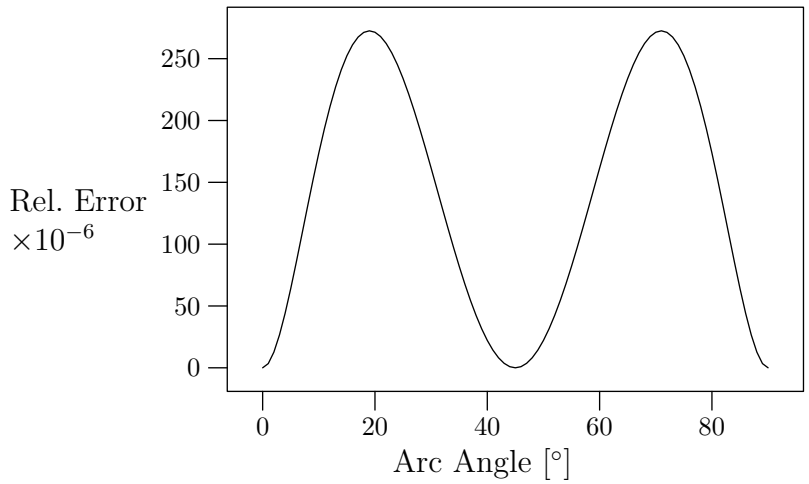


Circular arc parameter c defines curve tension. Derivation of proper c for given arc angle α.

$$
\begin{aligned}
z_0 &= (x_0, y_0) = (1, 0) && (1)\\
z_1 &= (x_1, y_1) = (1, c) && (2)\\
z_2 &= (x_2, y_2) = z_3 + (c\sin\alpha, -c\cos\alpha) && (3)\\
z_3 &= (x_3, y_3) = (\cos\alpha, \sin\alpha) && (4)\\
y_a &= \frac{y_1 + y_2}{2} = \frac{c + \sin\alpha - c\cos\alpha}{2} && (5)\\
y_b &= \frac{y_a + 0.5y_1}{2} = \frac{2c + \sin\alpha - c\cos\alpha}{4} && (6)\\
y_c &= \frac{y_a + \sin\alpha - 0.5c\cos\alpha}{2} = \frac{c + 3\sin\alpha - 2c\cos\alpha}{4} && (7)\\
y_d &= \frac{y_b + y_c}{2} = \frac{3c + 4\sin\alpha - 3c\cos\alpha}{8} \stackrel{!}{=} \sin\frac{\alpha}{2} && (8)\\
c &= \frac{8\sin(0.5\alpha) - 4\sin\alpha}{3(1 - \cos\alpha)} = \frac{4(1 - \cos(0.5\alpha))}{3\sin(0.5\alpha)} && (9)\\
c(\alpha = 90^\circ) &= \frac{4}{3}(\sqrt{2} - 1) = 0.55228\ldots && (10)
\end{aligned}
$$

This is the "magic number" mentioned on page 306 of [2].

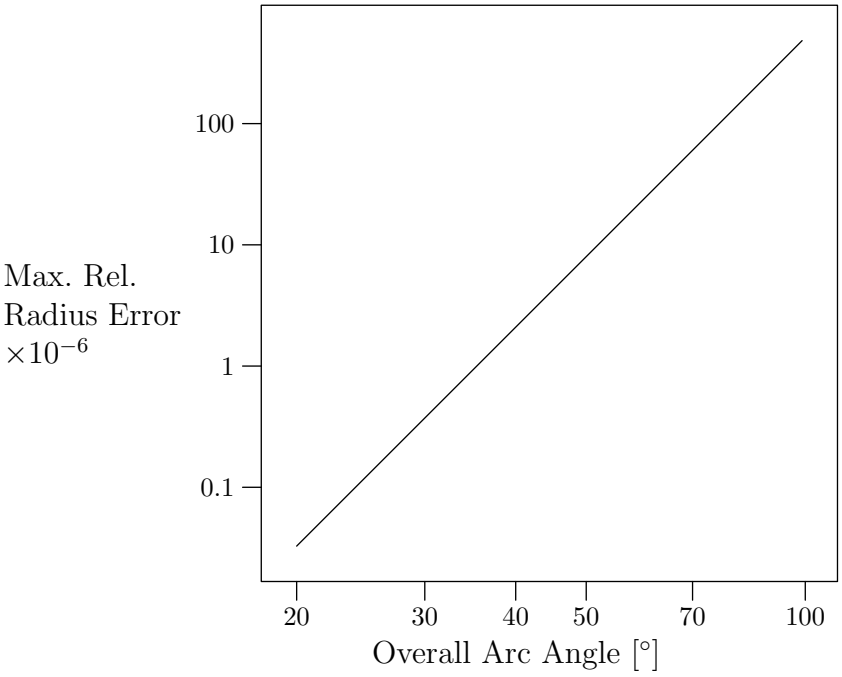
2 Relative Radius Error for 90 Degree Bézier Arc



Local Bézier curve radius is *larger* than exact radius (pos. rel. error). Max. relative error is 272.53×10^{-6} (1/3669 of radius).

3 Dependency of Max. Relative Radius Error from Overall Bézier Arc Angle

Plotted in log/log scale.



Rather straight curve, funny!

4 Disclaimer

All this is pretty trivial, written elsewhere long time ago. This note I wrote on a 'rainy' holiday, to check into Bézier curves, and as I couldn't get hold of the right book.

Updated 4 Dec. 2002.

References

[1] PostScript Language Reference Manual, 2nd ed. 1990, Adobe Systems Incorporated, Addison-Wesley, 13th printing, 1995.

[2] PostScript by Example, Henry McGilton, Mary Campione, 1992, Addison-Wesley, 2nd printing, 1994.